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A practical approach for monitoring process capability indices in the presence of autocorrelation

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Abstract

Autocorrelation is famous in continuous production processes, such as the processes in the chemical and pharmaceutical industries. In present time with the development of measurement technology and data acquisition technology, sampling frequency is getting higher and the existence of autocorrelation cannot be ignored. This paper gives a suitable method for different types of data for calculate process capability analysis in presence of auto correlated data.

Introduction

In statistics process control procedures are being used to control and maintain a satisfactory quality level and ensure that the proportion of defective items in the manufactured product is not too large. This termed as process control and is achieved through the technique of control charts, and the process capability indices are introduced to give a clear indication of the capability of a manufacturing process. They are formulated to quantify the relation between the desired engineering specifications and the actual performance of the process. The PCIs are organized to determine whether the process is capable of meeting specification limits on the quality features. The Quantitative measure of PCIs indicates the amount of Customers Requirements that are obtained from quality characteristics generally a large value of PCI shows a Better process. MAX-EWMA chart for auto correlated processes by Thaga and Yadavailli concentrate on the fact that positive auto correlation in the observation is capable of detecting changes in both process mean and standard deviation for auto correlation data. This chart is based on time series model to the data and the calculating thee residuals. Thee observation are represented as a first order auto regressive process plus a random error terms. The average run lengths for fixed decision intervals and reference values are calculated. The proposed chart is compared with the MAX-CUSUM chart for auto correlation data proposed by Thaga(2003) .In this paper there objective is to investigate control charts for simultaneously, monitoring the process mean and variation using a single chart in the presence of auto correlation. They propose an exponentially weighted moving average(EWMA) control chart for auto correlation data that can simultaneous monitor shifts in the mean and standard deviation using a single plotting variable. This investigation is done for the case of processes that can be modeled as a first order auto regressive AR(1) process plus an additional random error which can correspond to sampling, or measurement error. This model allows relatively accurate numerical techniques to be used to evaluate the properties of the control charts. EWMA control chart is capable of detecting changes positive auto correlation in observation can result in severe negative bias in traditional estimators of the standard deviation. this bias produces control limits that are much tighter than desired. This can result in a much higher average false alarm rate then expected. Furthermore, when observation are positively auto correlated, when there is a shift in the process mean, only a fraction of the shift will be transferred to the residual mean,and the chart will not quickly detect this shift. It is there fore very important to take auto correlation among observations into consideration when designing a process- monitoring scheme in particular, control charts- in order to maximize full benefits from the control charts.

Basic Indices

The most basic index.

(a) Unilateral with only U:

$$C_{pu} = \frac{U-\mu}{3\sigma} \qquad \text{; Provided that } \mu<=U$$

(b) Unilateral with only L:

$$C_{pl} = \frac{U-L}{3\sigma}$$; Provided that L <=µ

(c) Bilateral with = T=M

$$C_p = \frac{U-L}{6\sigma}$$

(d) Bilateral withT ≠ M

$$C_p^* = \min\{\frac{U-T}{3\sigma}, \frac{T-L}{3\sigma}\}$$

These indices are from Kane (1986). Cp is the most basic capability index and is said to be a first-generation index. The observant reader will immediately notice that Cp is the reciprocal of Juran & Gryna's capability ratio.

The Cpm index

The Cp and Cpk indices are appropriate measures of progress for quality improvement in which reduction of variability is the guiding principle and process yield is the primary measure of success. Taguchi (1986) has suggested a different approach to quality improvement in which reduction of variation from the target value is the guiding principle. In fact, Taguchi (1988) was the first author to propound the concept that there is a loss to society associated with missing the target. This concept of societal loss is difficult, if not impossible, to quantify. However, to be useful from a business perspective, a tool must be well defined, must be easy to use, and

must have a quantifiable financial impact so that results can be attributed to the success of the business. Taguchi realised that just being within specification is not sufficient, so he developed

the concept of the quadratic loss function to address the deficiency of the "goal post" approach to specification limits. In this approach, any measured value of a product characteristic X entails a monetary loss

L{x) to the customer as well as to the society in general. The loss function L is usually assumed to be well approximated by the symmetric squared error loss function. $L(x) = K(x-T)^2$

for some positive constant k, so that L(T) = 0; and any deviation from the ideal value entails some positive loss to the consumer or to the society. The capability of the process is represented by the expected loss function. $E(L) = KE\{(x-T)^{2\}}$

This is a measure of process variation in terms of deviation of the characteristic X from the target T. The appeal of expected loss is that it expresses process capability in monetary units, and therefore enters naturally into management decision-making process.

Schemes of Process Capability

Indices Estimation for Auto correlated Data There are two main methods, a model-based and a model-free approach, to deal with

Autocorrelations in the literature of process control. As the assessment of process capability begins after the evaluation of the process in a state of statistical control, five schemes are analyzed in the way similar to process monitoring with autocorrelation. MSE is used to make comparison among these model-based and model-free schemes. For $\hat{C}_{pm}$ it is defined as

$$MSE(\hat{C}_{pm}) = E(\hat{C}_{pr} - C_{pr})^2$$

$$VAR(\hat{C}_{pr}) + E[E(\hat{C}_{pr}) - C_{pr}]^2$$

One way to control auto correlated process is to adjust control limits of control chart in order to take non-random variation caused by autocorrelation into account based on process time series model. From the view point of methodology, it is realizable to adjust

Specification limit for process capability analysis with auto correlated data. However, in practice it may cause misunderstanding for the customers. Specification limits are usually established based on customers' requirements in order to determine whether the products are conforming. The application of specification limits is kept unchanged even for auto correlated processes. Thus, the way to adjust specification limits will not be discussed. Some schemes will be discussed.

Scheme with Observations

Using the formula $Cp = \frac{USL-LSL}{6\sigma}$ process capability can be assessed based on the statistics S^2 as the estimate of total process variance. This scheme is denoted as Scheme F_1, than $\hat{C}_{prF1}$, $\hat{C}_{pkrF1}$, $\hat{C}_{pmrF1}$, will be used to identify the indices $\hat{C}_{pr}, \hat{C}_{pkr}, \hat{C}_{pmr}$, with Scheme F_1. Indeed, a fitting model for process observations with autocorrelation is not necessary for the calculation of indices, but it is essential when analyzing confidence intervals of these indices.

Modified Scheme with Observations

For Scheme F1, auto correlated observations are applied directly to analyze process capability based on total process variability. Then it is

Considered to modify Scheme F1 with the extent to which a sample PCI is over- or under-estimating the true PCI, which is called modified scheme with observations, denoted as Scheme F2.

We use $\hat{C}_{pr}$ is an $\hat{C}_{prF2} = \hat{C}_{pr} * [\hat{C}_{pr} / E(\hat{C}_{pr})]$

Scheme with Residuals

For auto correlated process control, one of the most popular methods, called residual chart, is to monitor residuals which could be obtained

from the observations based on the process time series model. Using residuals, $\hat{\sigma}^2_e$ the variance of error in AR(1) model, could be estimated and be used for the assessment of process capability, which is denoted as Scheme F3.

$$\hat{C}_{prF3} = \frac{d}{3\hat{\sigma}_e}[1-/\hat{\phi}^2]^{1/2} \quad \text{……………………(1)}$$

$$\hat{C}_{pkrF3} = d - \frac{|\overline{X}-m|}{3\hat{\sigma}_e}[1-/\hat{\phi}^2]^{1/2} \quad \text{……………………………………(2)}$$

$$\hat{C}_{pmrF3} = \frac{d}{3\hat{\sigma}_e}\left(\frac{1-\phi^2}{1+k^2(1-/\hat{\phi}^2)}\right)^{1/2} \quad \text{………………………………...(3)}$$

where d=(USL- LSL)/2 and $k' = (\overline{X} - T)/\hat{\sigma}_e$

$\hat{\sigma}^2_e = (1-\hat{\phi}^2) * S^2$ then

MSE($\hat{C}_{prF3}$)=MSE($\hat{C}_{pr}$)

Leapingly Sampling Method

Leapingly sampling method is a method which subtracts sub-sample at intervals of time lf from total sample to meet the independence

assumption of process capability analysis. Based on observations of sub-sample, these indices are calculated directly using tradition definitions under the independence assumption. This method is denoted as Scheme F4, then $\hat{C}_{prF4}$, $\hat{C}_{pkrF4}$, and $\hat{C}_{pmrF4}$ will be used to identify the indices $\hat{C}_{pr}, \hat{C}_{pkr}, \hat{C}_{pmr}$ with Scheme F4. The essence of leapingly sampling method is to design sampling

interval l. The rule forbatch-mean method mentioned by Runger & Willemain (1995) could be used, i.e. to select an l s.t. $\rho < 0.1$ In AR(1) model,
$\rho_i = \phi^i$ When $\phi = 0.9$ and l=22 It appears that the determination of sampling interval l requires knowledge of autocorrelation coefficient. The rule in the literature discussed above is to have a sampling interval l larger than 20 or even larger when data are sufficient. Obviously a large amount of data is absolutely necessary in this method to ensure sufficient sub-samples and reliable assessment for process capability.

Conclusion

Test if the data are normally distributed. Methods include Shapiro-Wilk, Kolmogorov-Smirnov, Cramer-von Mises and Anderson-Darling. Test if the process is in state of statistical control. If the data are correlated, auto correlated coefficient and partial correlation coefficient are used to determine the derivative order of the characteristics of autocorrelation model. Some control charts corresponding to auto correlated processes, such as residual control chart, the autoregressive chart (Apley & Tsung, 2002), batch-means control chart (Runger & Willemain, 1996) and so on, could be used to test if the process is in state of statistical control.

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